# Corporate Finance

## Discounted Cash Flow Valuation

Chapter 5

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# Multiple Cash Flows

## Example (Future Value)

You think you will be able to deposit \$4,000 at the end of each of the next three years in a bank account paying 8 percent interest. You currently have \$7,000 in the account. How much will you have in 3 years? How much in 4 years?

Find the value at year 3 of each cash flow and add them together.

- Year 0: $FV = \$7,000(1.08)^3 = \$8,817.98$
- Year 1: $FV = \$4,000(1.08)^2 = \$4,665.60$
- Year 2: $FV = \$4,000(1.08)^1 = \$4,320.00$
- Year 3: $Value = \$4,000.00$
- Total value in 3 years \$21,803.58
- Value at year 4 is $\$21,803.58(1.08) = \$23,547.87$.

## Example (Future Value: Uneven Multiple Cash Flows)

If you deposit $100 in one year, $200 in two years and $300 in three years. How much will you have in three years at 7 percent interest? How much in five years if you don't add additional amounts?

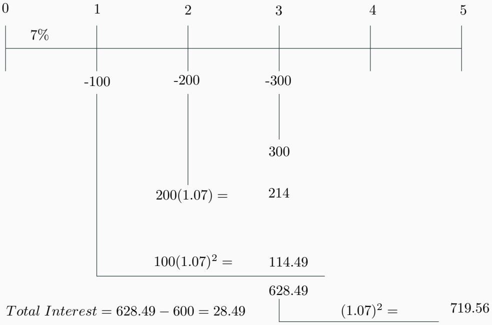


**Figure 1:** Multiple Uneven Cash Flow

# Multiple Cash Flows Cont'd

### Example (Future Value: Uneven Multiple Cash Flows cont'd)

| Year | NPER | CF | FV | Function |
|---|---|---|---|---|
| 1 | 2 | -100 | 114.49 | $FV(0.07, 2, 0, -100)$ |
| 2 | 1 | -200 | 214 | $FV(0.07, 1, 0, -200)$ |
| 3 | 0 | -300 | 300 | $FV(0.07, 0, 0, -300)$ |

- Total $FV$ at Year 3: \$628.49
- Total $FV$ at Year 5: $\$719.56 = 628.49(1.07)^2$

# Multiple Cash Flows Cont'd

## Example

Suppose you plan to deposit $100 into an account in one year and $300 into the account in three years. How much will be in the account in five years if the interest rate is 8%?

$$FV = 100(1+0.08)^4 + 300(1+0.08)^2 = 136.05 + 349.92 = 485.97.$$

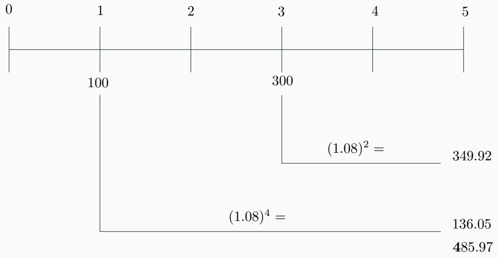


**Figure 2:** Multiple Uneven Cash Flow

# Multiple Cash Flows Cont'd

## Example (Present Value)

You are considering an investment that will pay you $1,000 in one year, $2,000 in two years and $3,000 in three years. If you want to earn 10% on your money, how much would you be willing to pay?

$$
\begin{aligned}
PV =& \frac{1,000}{1.1} = 909.09 \\
=& \frac{2,000}{(1.1^2} = 1,652.89 \\
=& \frac{3,000}{(1.1)^3} = 2,253.94 \\
PV_{\sum=}& 4,815.92
\end{aligned}
$$

# Annuities and Perpetuities

- Annuity is defined as finite series of equal payments that occur at regular intervals.
  - If the first payment occurs at the end of the period, it is called an *ordinary annuity*;
  - If the first payment occurs at the beginning of the period, it is called an *annuity due*.
- Perpetuity is defined as infinite series of equal payments.

# Annuities and Perpetuities Cont'd

- Basic Formulas
  - Perpetuity

$$PV = \frac{PMT}{r}$$

  - Annuities

$$PV = PMT \left[ \frac{1 - \frac{1}{(1+r)^t}}{r} \right]$$

$$FV = PMT \left[ \frac{(1+r)^t - 1}{r} \right]$$

# Excel Spreadsheet Functions

- $FV(Rate, nper, pmt, pv, 0/1)$;
- $PV(Rate, nper, pmt, fv, 0/1)$;
- $RATE(nper, pmt, pv, fv, 0/1)$;
- $NPER(Rate, pmt, pv, fv, 0/1)$;
- $PMT(Rate, nper, pv, fv, 0/1)$.
    - Ordinary Annuity is 0 (default; no entry needed);
    - Annuity Due is 1 (must be entered).

# Annuities and Perpetuities Cont'd

- Important Points to Remember
  - Interest rate and time period must match:
    - Annual periods with annual rate;
    - Monthly periods with monthly rate.
  - The Sign Convention
    - Cash inflows are positive;
    - Cash outflows are negative.

## Example

You can afford \$632 per month. Going rate is 1% monthly for 48 months. How much can you borrow?

You borrow money today so you need to compute the present value:

$$PV = 632 \left[ \frac{1 - \frac{1}{(1.01)^{48}}}{0.01} \right] = 23,999.54$$

Excel Solution: $PV(0.01, 48, -632, 0)$

# Annuity Due

## Example

You are saving for a new house and you put $10,000 per year in an account paying 8%. The first payment is made today. How much will you have at the end of 3 years?

$$\begin{aligned} FV =& PMT\left[\frac{(1+r)^t - 1}{r}\right](1+r) \\ =& 10,000\left[\frac{(1.08)^3 - 1}{0.08}\right](1.08) = 35,061.12 \end{aligned}$$

Excel Solution:

$$FV(0.08, 3, -10000, 0, 1)$$

# Interest Rates

- Effective Annual Rate (EAR)
  - The interest rate expressed as if it were compounded once per year;
  - Used to compare two alternative investments with different compounding periods.
- Annual Percentage Rate (APR) 'Nominal'
  - The annual rate quoted by law;
  - *APR* is the periodic rate *X* number of periods per year.

$$Periodic\ Rate = \frac{APR}{Periods\ per\ Year}$$

- If you have an *APR* based on monthly compounding, you have to use monthly periods for lump sums or adjust the interest rate accordingly.

# *EAR* Formula

- *EAR* is given by

$$EAR = \left[1 + \frac{APR}{m}\right]^m - 1$$

where,

  - *APR* is the quoted rate; and
  - *m* is the number of compounds per year.

**Example**

Which savings accounts should you choose:

- 5.25% with daily compounding;

$$EAR = (1 + 0.0525/365)^{365} - 1 = 5.39\%$$

$$EFFECT(0.525, 365) = 5.39\%$$

- 5.30% with semi-annual compounding.

$$EAR = (1 + 0.053/2)^2 - 1 = 5.37\%$$

$$EFFECT(0.53, 2) = 5.37\%$$

# Computing A*PR*s

## Example

What is the APR if the monthly rate is .5%?

$$0.05(12) = 6\%$$

What is the APR if the semi-annual rate is .5%?

$$0.05(2) = 1\%$$

What is the monthly rate if the APR is 12% with monthly compounding?

$$\frac{0.12}{12} = 1\%$$

Can you divide the above *APR* by 2 to get the semi-annual rate? No. You need an APR based on semi-annual compounding to find the semi-annual rate.

# Computing EAR and APR

## Example

Suppose you can earn 1% per month on $1 invested today. What is the APR?

$$1(12) = 12\%$$

How much are you effectively earning?

$$FV = 1(1.01)^{12} = 1.1268$$
$$Rate = \frac{(1.1268 - 1)}{1} = 0.1268 = 12.68\%$$

Excel Solution:

$$EFFECT(0.12, 12)$$

# Computing EAR and APR Cont'd

### Example

Suppose if you put it in another account, you earn 3% per quarter. What is the APR?

$$3(4) = 12\%$$

How much are you effectively earning?

$$FV = 1(1.03)^4 = 1.1255$$

$$Rate = \frac{(1.1255 - 1)}{1} = 0.1255 = 12.55\%$$

Excel Solution:

$$EFFECT(0.12, 4)$$

# Computing *APR*s from *EAR*s

- Let *APR* be

$$APR = m\left[(1+EAR)^{1/m} - 1\right]$$

where, $m$ is the number of compounding periods per year.

**Example**

Suppose you want to earn an effective rate of 12% and you are looking at an account that compounds on a monthly basis. What APR must they pay?

$$APR = 12\left[(1+0.12)^{1/12} - 1\right] = 0.1138655 = 11.39\%$$

Excel Solution:

$$NOMINAL(0.12, 12)$$

# Computing Payments with *APR*s

## Example

Suppose you want to buy a new computer. The store is willing to allow you to make monthly payments. The entire computer system costs \$3,500. The loan period is for 2 years. The interest rate is 16.9% with monthly compounding. What is your monthly payment?

Excel Solution:

$$2(12) = 24,$$
$$\frac{16.9}{12} = 1.40833,$$
$$PMT(0.0140833, 24, 3500, 0) = 172.88$$

# Future Values with Monthly Compounding

## Example

Suppose you deposit \$50 a month into an account that has an APR of 9%, based on monthly compounding. How much will you have in the account in 35 years?

Excel Solution:

$$
\begin{aligned}
N =& 35(12) = 420 \\
\frac{9}{12} =& 0.75 \\
PV =& 0 \\
PMT =& -50 \\
=& FV(0.0075, 420, -50, 0) = 147,089.22
\end{aligned}
$$

# Present Value with Daily Compounding

## Example

You need $15,000 in 3 years for a new car. If you can deposit money into an account that pays an APR of 5.5% based on daily compounding, how much would you need to deposit?

Excel Solution:

$$
\begin{aligned}
N =& 3(365) = 1095 \\
\frac{5.5}{365} =& 0.015068493 \\
PMT =& 0 \\
FV =& 15,000 \\
=& PV(0.00015, 1095, 0, 15000) = 12,728.10
\end{aligned}
$$

# Pure Discount Loans

- Treasury bills are excellent examples of pure discount loans.
  - Principal amount is repaid at some future date;
  - No periodic interest payments.

## Example (Amortized Loan with Fixed Payment)

*Each payment covers the interest expense plus reduces principal*. Consider a 4-year loan with annual payments. The interest rate is 8% and the principal amount is \$5000. What is the annual payment?

$$5,000 = \frac{PMT\left[1 - \frac{1}{(1.08)^4}\right]}{0.08}$$

$$PMT = 1,509.60$$

Excel Solution:

$$PMT(0.08, 4, 5000, 0) = 1509.60.$$

# Pure Discount Loans Cont'd

## Example (Amortized Loan with Fixed Payment)

| Year | Beginning Balance | Total Payment Payment | Interest Paid | Principal Paid | Ending Balance |
|---|---|---|---|---|---|
| 1 | $ 5,000.00 | $ 1,509.60 | $ 400.00 | $ 1,109.60 | $ 3,890.40 |
| 2 | $ 3,890.40 | $ 1,509.60 | $ 311.23 | $ 1,198.37 | $ 2,692.03 |
| 3 | $ 2,692.03 | $ 1,509.60 | $ 215.36 | $ 1,294.24 | $ 1,397.79 |
| 4 | $ 1,397.79 | $ 1,509.60 | $ 111.82 | $ 1,397.79 | $ - |
| Totals | | $ 6,038.40 | $ 1,038.42 | $ 5,000.00 | |

**Figure 3:** Amortized Loan with Fixed Payment

$$\begin{aligned} \textit{Interest Paid} &= \textit{Beginning Balance} \times \textit{Rate}(8\%) \\ \textit{Principal Paid} &= \textit{Total Payment} - \textit{Interest Paid} \\ \textit{Ending Balance} &= \textit{Beginning Balance} - \textit{Principal Paid} \end{aligned}$$